MODELING OF CAPILLARY PRESSURE HYSTERESIS AND OF HYSTERESIS OF RELATIVE PHASE PERMEABILITIES IN POROUS MATERIALS ON THE BASIS OF THE PORE ENSEMBLE CONCEPT

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We propose a class of models of porous media based on the pore ensemble concept with a certain distribution of the main geometric parameters (e.g., pore size). The case of the saturation of the pore space with a two-phase liquid mixture has been considered. The transfer laws are deduced from the free energy dissipation condition. The hydrodynamic connection of pores is described by two kinds of kernels: one kernel describes the connection of pores in the space, and the other kernel describes the connection of pores in an elementary macrovolume. A simple model of capillary pressure hysteresis associated with the monotonicity of the function of the area-to-volume ratio of the pore is proposed. The results of numerical calculations of the capillary pressure hysteresis and relative phase permeabilities are presented.

Introduction. At present, the main means for mathematical description of the processes in porous materials is the continuum model where both the porous material itself and the liquid contained in its pores are assumed to be continuously distributed in the space [1–6], and individual properties of the pores are assumed to be immaterial and it suffices to take these properties into account by means of the macroscopic porosity and permeability coefficients.

At the same time the experimentally observed dynamic effects concerning the absolute and phase permeability coefficients, the capillary pressure, and the impurity transfer led to the appearance of a large number of models taking into account the structure of pores, e.g., capillary aggregation models or mesh models (see review [7]). Note that the size distribution of pores is an important component of the colmation theory [8]. However, each of the developed models of the pore space structure was intended for solving a quite definite particular problem — the problem of calculating the phase permeabilities, capillary pressure, etc.

In the present work, we propose a general mathematical formalism for models based on the pore ensemble concept. It has been shown that this approach permits modeling the capillary pressure hysteresis and the hysteresis of phase permeabilities.

1. Main Concepts of the Pore Ensemble Model. Two-Phase Case. Consider an isotropic homogeneous rigid porous medium. All processes are assumed to be isothermal and, therefore, the temperature dependence is omitted everywhere. It is also assumed that the gravitational forces are immaterial. Indices a, b, c run through the values of 1, 2, 3 corresponding to the ordinal numbers of spatial Cartesian coordinates x^a. Summation is made over recurring spatial indices. Abridged notations for partial derivatives $\partial_a = \partial/\partial x^a$, $\partial_t = \partial/\partial t$, where t is time, are used.

The first step in defining the pore ensemble is the introduction of the collective parameter ζ which enumerates or indexes various pores in a selected elementary macrovolume. This parameter can incorporate the characteristic size or other geometric properties of a pore, the pore index in the case of pores of different types (e.g., for a jointing-porous rock), or simply coincide with the pore number.

The second step is to give the probability measure $\partial\mu(\zeta)$ on the pore space characterizing the frequency of occurrence of a pore ζ in the ensemble. By definition, the normalization condition

$$\int d\mu\,(\zeta) = 1 \tag{1}$$

takes place.

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Further, for each pore of ζ the pore volume $\upsilon = \upsilon(\zeta)$ and the pore surface area $\sigma = \sigma(\zeta)$ are introduced. These quantities are assumed to be normalized to a unit volume so that their integration with respect to the probability measure gives the macroscopic porosity ϕ and the specific surface S:

$$\phi = \int \upsilon\,(\zeta)\,d\mu\,(\zeta)\,, \tag{2}$$

$$S = \int \sigma\,(\zeta)\,d\mu\,(\zeta)\,. \tag{3}$$

Consider a situation where in the pores two immiscible phases (liquid or gas) are present: phase A and phase B (this can be, for example, an oil–water or water–gas mixture). Let us give in each pore the mass densities of the phases $\rho_A = \rho_A(t, x^a, \zeta)$, $\rho_B = \rho_B(t, x^a, \zeta)$ and saturations of the phases $\omega_A = \omega_A(t, x^a, \zeta)$, $\omega_B = \omega_B(t, x^a, \zeta)$, where $0 \le \omega_A, \omega_B \le 1$, $\omega_A + \omega_B = 1$. The corresponding macroscopic saturations of the phases $s_A(t, x^a)$, $s_B(t, x^a)$ and the macroscopic mass densities $r_A(t, x^a)$, $r_B(t, x^a)$ are calculated by averaging over the pore ensemble

$$s_\Lambda\,(t, x^a) = \phi^{-1} \int \omega_\Lambda\,(t, x^a, \zeta)\,\upsilon\,(\zeta)\,d\mu\,(\zeta)\,, \tag{4}$$

$$r_\Lambda\,(t, x^a) = \phi^{-1}\,s_\Lambda\,(t, x^a)^{-1} \int \omega_\Lambda\,(t, x^a, \zeta)\,\rho_\Lambda\,(t, x^a, \zeta)\,\upsilon\,(\zeta)\,d\mu\,(\zeta)\,, \tag{5}$$

where Λ = A. B.

In the absence of phase transitions the dynamics of the mass transfer between the pores is described by the differential transfer equation for each phase

$$\partial_t\,(\upsilon\omega_\Lambda\rho_\Lambda) + \partial_a i_\Lambda^a = j_\Lambda\,, \quad \Lambda = \text{A, B}\,, \tag{6}$$

where $i_\Lambda^a = i_\Lambda^a(t, x^b, \zeta)$ is the three-dimensional flow of phase Λ (A or B): $j_\Lambda = j_\Lambda(t, x^b, \zeta)$ is the inflow of the phase Λ to the pore ζ from the other pores in the same elementary macrovolume. The flows between the pores inside the elementary macrovolume should satisfy the conditions of mass conservation

$$\int j_\Lambda\,(t, x^a, \zeta)\,d\mu\,(\zeta) = 0\,, \quad \Lambda = \text{A, B}\,. \tag{7}$$

In view of relations (7) and definitions (4), (5) integration of the conservation equations (6) with respect to the pore ensemble leads to the macroscopic laws of conservation

$$\partial_t\,(\phi s_\Lambda r_\Lambda) + \partial_a I_\Lambda^a = 0\,, \quad \Lambda = \text{A, B}\,, \tag{8}$$

where the macroscopic flows I_A^a, I_B^a are defined by the following expressions:

$$I_\Lambda^a\,(t, x^a) = \int i_\Lambda^a\,(t, x^a, \zeta)\,d\mu\,(\zeta)\,, \quad \Lambda = \text{A, B}\,. \tag{9}$$

Problem (6) (as problem (8)) is nonclosed — the number of unknowns exceeds the number of equations. To obtain a closed system of equations, additional relations are needed. Such relations can be derived by testing the model for compliance with the second law of thermodynamics, which for isothermal processes is reduced to the free energy dissipation condition (the dissipativity condition).

Let $f_A = f_A(\rho_A)$ ($f_B = f_B(\rho_B)$) be the free energy of the phase A (phase B) mixture per unit volume. The temperature dependence is omitted. It will be recalled that the free energy function satisfies the following thermodynamic relations [9]:

$$\kappa_\Lambda = \frac{\partial f_\Lambda}{\partial \rho_\Lambda}\,, \quad p_\Lambda = -f_\Lambda + \rho_\Lambda \kappa_\Lambda\,, \quad dp_\Lambda = \rho_\Lambda d\kappa_\Lambda\,, \tag{10}$$

where $\Lambda = A, B$; κ_Λ is the chemical potential in the phase Λ; p_Λ is the hydrostatic pressure in the phase Λ.

The free energy of the two-phase mixture $H = H(t, x^a)$ in the elementary macrovolume of the porous medium is composed of the free energy in the volume $H_v = H_v(t, x^a)$ and the surface energy $H_s = H_s(t, x^a)$:

$$H = H_v + H_s , \tag{11}$$

$$H_v = \int (\omega_A f_A (\rho_A) + \omega_B f_B (\rho_B)) \upsilon d\mu (\zeta) ,$$

$$H_s = \int (\omega_A \gamma_A + \omega_B \gamma_B) \sigma d\mu (\zeta) + \gamma_{AB} \int \Omega (\zeta, \zeta') \omega_A \omega_B' d\mu (\zeta) \, d\mu (\zeta') ,$$

where $\omega_B' = \omega_B(t, x^a, \zeta')$; $\Omega = \Omega(\zeta, \zeta')$ is the symmetrical (probably generalized) function characterizing the contact surface between pores ζ, ζ'. From Eqs. (6), (10), (11) there follows the evolution equation for the free energy

$$\partial_t H + \partial_a H^a = \Sigma , \quad H^a = \int \left(i_A^a \kappa_A + i_B^a \kappa_B \right) d\mu (\zeta) , \tag{12}$$

$$\Sigma = \int \left(i_A^a \partial_a \kappa_A + i_B^a \partial_a \kappa_B + j_A \kappa_A + j_B \kappa_B + \partial_t \omega_A (\psi_A - \psi_B) \right) d\mu (\zeta) ,$$

$$\psi_A = - \upsilon p_A + \sigma \gamma_A + \gamma_{AB} \int \Omega (\zeta, \zeta') \omega_B' d\mu (\zeta') ,$$

$$\psi_B = - \upsilon p_B + \sigma \gamma_B + \gamma_{AB} \int \Omega (\zeta, \zeta') \omega_A' d\mu (\zeta') .$$

Here H^a is the three-dimensional flow of the free energy; Σ denotes the free energy production.

The dissipativity condition of the model means the fulfilment of the inequality $\Sigma \leq 0$, to satisfy which we assume the following constitutive relations in the elementary macrovolume:

$$i_\Lambda^a = - \mu_\Lambda^{-1} \rho_\Lambda \omega_\Lambda \int K \rho_\Lambda' \omega_\Lambda' \partial_a \kappa_\Lambda' d\mu (\zeta') , \tag{13}$$

$$j_\Lambda = \mu_\Lambda^{-1} \rho_\Lambda \omega_\Lambda \int D \rho_\Lambda' \omega_\Lambda' (\kappa_\Lambda' - \kappa_\Lambda) \, d\mu (\zeta') , \tag{14}$$

where $\Lambda = A, B$; μ_Λ denotes the shear viscosities of the phases which are assumed to be constant: $\kappa_\Lambda' = \kappa_\Lambda(t, x^b, \zeta')$; $\rho_\Lambda' = \rho_\Lambda(t, x^b, \zeta')$; $K = K(\zeta, \zeta')$ is the symmetrical kernel describing the connection of pores in the space; $D = D(\zeta, \zeta')$ is the symmetrical kernel describing the connection of pores in the elementary macrovolume. The integral operators specified by kernels $K = K(\zeta, \zeta')$, $D = D(\zeta, \zeta')$ are assumed to be positive. Apart from relations (13), (14) for pores with a nonzero saturation of both phases ($0 < \omega_A$, $\omega_B < 1$), let us take the condition equivalent to the instantaneous establishment of the capillary pressure:

$$p_B - p_A = p_{cap} , \tag{15}$$

$$p_{cap} = (\gamma_B - \gamma_A) \sigma \upsilon^{-1} + \gamma_{AB} \upsilon^{-1} \int \Omega (\omega_A' - \omega_B') \, d\mu (\zeta') . \tag{16}$$

It is seen that relations (13)–(16) provide the fulfilment of the inequality $\Sigma \leq 0$ and, therefore, the model constructed is thermodynamically correct. For the case of small changes in the chemical potentials κ_A, κ_B, we can make use of the Gibbs–Duhem relation (the second relation of (10)) and transform expressions (13), (14) to the form

$$i_\Lambda^a = - \mu_\Lambda^{-1} \rho_\Lambda \omega_\Lambda \int K \omega_\Lambda' \partial_a p_\Lambda' d\mu (\zeta') , \tag{17}$$

$$j_\Lambda = \mu_\Lambda^{-1} \rho_\Lambda \omega_\Lambda \int D \omega_\Lambda' (p_\Lambda' - p_\Lambda) \, d\mu (\zeta') , \tag{18}$$

where $p_\Lambda' = p_\Lambda(t, x^a, \zeta')$.

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Substituting relations (17) into (9) permits calculating the macroscopic flows. In the particular case where the pressure values in different pores at one point in the space coincide but there is a pressure gradient in the space, from (17) and (9) there follows the modified Darcy law for the two-phase flow with phase permeabilities k_A, k_B:

$$I_\Lambda^a = -\mu_\Lambda^{-1} \rho_\Lambda k_\Lambda \partial_a p_\Lambda \,, \tag{19}$$

$$k_\Lambda = \int K \omega_\Lambda \omega_\Lambda' d\mu\,(\zeta)\, d\mu\,(\zeta') \,, \quad \Lambda = A,\ B \,. \tag{20}$$

As follows from (16), (20), in the general case the capillary pressure and the phase permeabilities depend on the saturation distribution over the pore ensemble. However, of practical interest are situations where such a distribution is close to equilibrium. For the equilibrium distribution of the two phases over the pore ensemble, flows (18) go to zero, which corresponds to the equality of the chemical potentials in different pores $\kappa_A' = \kappa_A$, $\kappa_B' = \kappa_B$ and, consequently, to the equality of pressures $p_A' = p_A$, $p_B' = p_B$. Thus, the value of the capillary pressure jump p_{cap} in expressions (15), (16) does not depend on the parameter ζ. Therefore, (16) is, in essence, an equation defining the saturation distribution at a given capillary pressure jump p_{cap}.

For a more concrete consideration of equilibrium distributions, let us take the expression for the phase contact in the pores in the form $\Omega(\zeta, \zeta') = \theta(\zeta)\delta(\zeta, \zeta')$, where $\delta(\zeta, \zeta')$ is a Dirac delta function; $\theta(\zeta)$ is a positive function. Then the equilibrium conditions (free energy minimum conditions (11)) for the saturation distribution at fixed p_{cap} are as follows: at $\psi_A > \psi_B$ $\omega_A = 0$, at $\psi_A < \psi_B$ $\omega_A = 1$, and at $\psi_A = \psi_B$ $0 \le \omega_A \le 1$. These cases are not independent. Upon substitution of the expression for $\Omega(\zeta, \zeta')$ and introduction of the dimensionless auxiliary function

$$\Psi = \Psi\,(\zeta) = \left[p_{cap}\upsilon\,(\zeta) + (\gamma_A - \gamma_B)\,\sigma\,(\zeta) \right] \left[\gamma_{AB}\theta\,(\zeta)\,\upsilon\,(\zeta) \right]^{-1} \tag{21}$$

the free energy minimum conditions (11) take on the form: at $\psi(\zeta) < -1$ $\omega_A(\zeta) = 1$, at $\psi(\zeta) > 1$ the saturation $\omega_A(\zeta) = 0$, and at $\left| \psi(\zeta) \right| < 1$ $\omega_A(\zeta) = 0$ or $\omega_A(\zeta) = 1$. In the last case, both the zero and the unit saturations provide a free energy minimum.

Thus, there is ambiguity in the definition of the equilibrium saturation distribution caused by the interphase interaction. Moreover, there is one more source of ambiguity — the possible nonmonotonicity of the function

$$\Phi\,(\zeta) = (\gamma_B - \gamma_A)\,\sigma\,(\zeta)\,\upsilon\,(\zeta)^{-1} \,. \tag{22}$$

Indeed, suppose that the interchange of matter can occur only between pores having a close value of the parameter ζ. Let, in addition, the set of pores $\psi(\zeta) > 1$ (or the set of pores $\psi(\zeta) < -1$) decompose into disconnected subsets. Such a situation is possible for the nonmonotonic function (22). And the filling of these disconnected subsets with different phases will provide a local minimum of free energy, but it will not be global.

The ambiguity in the saturation distribution leads to the fact that to one value of the capillary pressure more than one value of the macroscopic saturation can be assigned. For different scenarios of the displacement of one phase by the other the above effect will show up as a capillary pressure hysteresis.

2. Computational Modeling of the Capillary Hysteresis. To demonstrate the possibility of describing the hysteresis in the pore ensemble theory, we have performed a computational modeling of this phenomenon on two simple models described below. It was assumed that the influence of the term in the expression for the surface energy associated with the kernel $\Omega(\zeta, \zeta')$ is negligibly small. Then the expression for the capillary pressure (16) takes the form $p_{cap} = \Phi(\zeta)$, and the equilibrium saturations of the phases in the pores can take only the values 0 or 1.

Now, at any fixed value of the capillary pressure p_{cap}^* we can calculate the distribution of the phases between the pores, after which, according to (4), we can calculate the macroscopic saturations. It should be emphasized that all pores $\Phi(\zeta) > p_{cap}^*$ turn out to be filled with the wetting phase, and pores $\Phi(\zeta) < p_{cap}^*$ — with the nonwetting phase only if free interchange of matter for all pores is possible. As was shown in Subsection 1, the disconnection of the set of pores $\Phi(\zeta) > p_{cap}^*$ (or the set of pores $\Phi(\zeta) < p_{cap}^*$) leads to an ambiguous filling of pores with the phases at a fixed capillary pressure and, consequently, to a hysteresis.
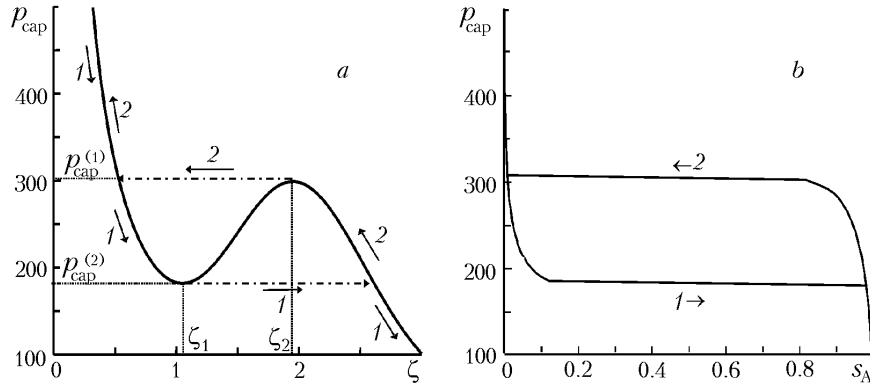
Fig. 1. Capillary pressure p_{cap} versus the one-dimensional parameter ζ (a) and capillary pressure hysteresis as a function of the wetting phase saturation s_A (b) for the processes of impregnation (1) and drainage (2). p_{cap}, Pa; ζ, μm.

Example 1. The one-dimensional parameter ζ runs through the real positive values and has the meaning of the characteristic size of pores. If the function $\sigma(\zeta)\upsilon(\zeta)^{-1}$ is monotonic, then the set of pores $\Phi(\zeta) > p^*_{cap}$ (or the set of pores $\Phi(\zeta) < p^*_{cap}$) is always connected (it represents an interval on the real axis) and a capillary hysteresis is absent. We investigated a model with the nonmonotonic function $\sigma(\zeta)\upsilon(\zeta)^{-1}$:

$$d\mu(\zeta) = g(\zeta)\,d\zeta, \quad g(\zeta) = 2\zeta l^{-2}\exp(-\zeta^2 l^{-2}),$$

$$\upsilon = \upsilon(\zeta) = \phi\Xi\left(\frac{\zeta}{l}\right)^3, \quad \Xi = \frac{4}{3}\pi^{-1/2},$$

$$\sigma = \sigma(\zeta) = S\Theta^{-1}\left(\frac{\zeta}{l}\right)^2\left(1 + G\left(\frac{\zeta}{l}\right)^Z\exp(-\alpha l^{-2}\zeta^2)\right),$$

$$\Theta = 1 + (1+\alpha)^{-(2+0.5Z)}G\Gamma(2+0.5Z), \tag{23}$$

where Γ is the gamma function; $\varphi = 0.18$; $S = 46{,}476$ m^{-1} is the specific surface; $l = 1$ μm; $\alpha = 0.87$; $G = 0.35$; $Z = 8$. The coefficients Ξ and Θ were chosen proceeding from the normalization condition (2), (3). The following surface tension coefficients were taken: $\gamma_A = -0.008$ N/m (the phase A is wetting), $\gamma_B = 0.003$ N/m (the phase B is nonwetting).

In calculating the relative phase permeabilities, we assumed the following form of the kernel $K(\zeta, \zeta')$ describing the connection of pores in the space:

$$K(\zeta, \zeta') = K(\zeta)\,\delta(\zeta - \zeta'), \quad K(\zeta) = k_0\Pi^{-1}\left(\frac{\zeta^2}{l^2}\right), \tag{24}$$

here $k_0 = 1$ μm^2 is the absolute permeability. The coefficient Π was chosen proceeding from the normalization condition $k_0 = \int K(\zeta, \zeta')d\mu(\zeta)d\mu(\zeta')$.

The results of the calculations of the capillary pressure and phase permeabilities are presented in Figs. 1 and 2. It is assumed that pores can interchange matter only with pores having a close value of ζ. In the process of impregnation under the action of the capillary pressure drop (hereinafter spontaneous impregnation [7] when $p_{cap} > 0$ is considered), the wetting phase gradually penetrates from small pores into large ones (characterized by a larger value of the parameters ζ and a lower capillary pressure). The nonmonotonicity of the function σ/υ leads to a nonmonotonic dependence of the capillary pressure on the radius of pores (Fig. 1a). Accordingly, when pores with a characteristic
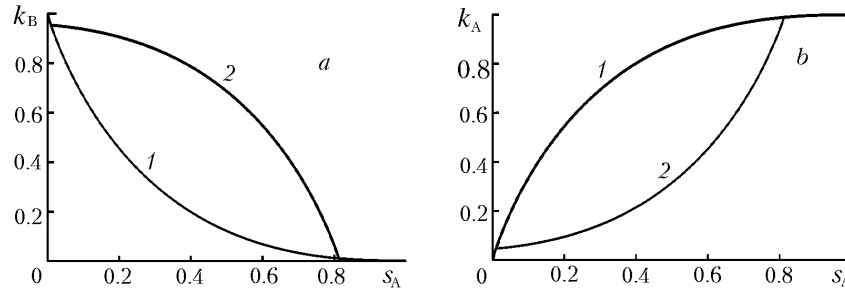
Fig. 2. Hysteresis of the relative phase permeability of the nonwetting (a) and wetting (b) phases as a function of the wetting phase saturation s_A for the one-dimensional parameter ζ: 1) impregnation; 2) drainage.
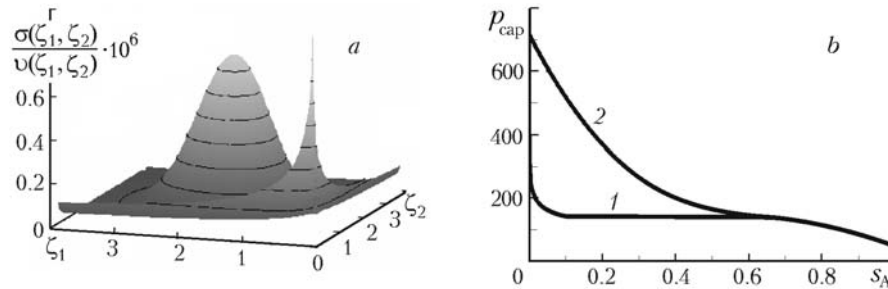

Fig. 3. Function of the area-to-volume ratio of the pore $\sigma(\zeta_1, \zeta_2)/\upsilon(\zeta_1, \zeta_2)$ (a) and capillary pressure hysteresis as a function of the wetting phase saturation s_A (b) for the processes of impregnation (1) and drainage (2) in the case of the two-dimensional parameter $\zeta(\zeta_1, \zeta_2)$. p_{cap}, Pa; ζ_1, ζ_2, μm.

size ζ_1 are filled with the wetting phase, some of the large pores ($\zeta > \zeta_1$) have a higher capillary pressure than $p_{cap}^{(2)}$ — the pressure in pores of radius ζ_1. These pores will be further filled with the wetting phase with no change in the capillary pressure (dash-and-dot line 1 in Fig. 1a). This process corresponds to the straight-line portion of the impregnation curve (Fig. 1b).

In the process of drainage, the nonwetting phase is injected into a porous specimen under the excess ambient pressure. With increasing pressure applied the nonwetting phase penetrates from large pores into small ones having a higher value of the capillary pressure. By virtue of the nonmonotonic dependence of the capillary pressure on the radius of pores (Fig. 1a), when pores of radius ζ_2 are filled with the nonwetting phase, in some of the pores with a smaller radius the capillary pressure turns out to be lower than $p_{cap}^{(1)}$ — the pressure in pores of radius ζ_2. These pores will be further filled with the nonwetting phase with no change in the capillary pressure (dash-and-dot line 2 in Fig. 1a). This process corresponds to the straight-line portion on the drainage curve (Fig. 1b). The difference between the capillary pressures of pores of radius ζ_1 and ζ_2 ($p_{cap}^{(1)} \neq p_{cap}^{(2)}$) leads to the formation of a hysteresis formed by the drainage and impregnation curves.

The calculations of the capillary pressure agree, in general, with the experiment [7, 10]. However, the experimental curves of the capillary pressure for the drainage and impregnation processes are sloping, and the calculation curves are parallel to the abscissa axis in a wide range of change in the saturation. A more exact description of the capillary hysteresis is attained in the following example.

Example 2. The parameter $\zeta = (\zeta_1, \zeta_2)$ is a two-dimensional vector, and ζ_1 and ζ_2 therewith run through the real positive values. The variables ζ_1, ζ_2 can represent, e.g., the principal radii of inertia of pores. A model with the nonmonotonic function $\sigma(\zeta_1, \zeta_2)\upsilon^{-1}(\zeta_1, \zeta_2)$ was given:

$$d\mu(\zeta) = g(\zeta_1)\, g(\zeta_2)\, d\zeta_1 d\zeta_2\,, \quad g(\zeta_i) = 2\zeta_i l^{-2} \exp\left(-\zeta_i^2 l^{-2}\right),$$
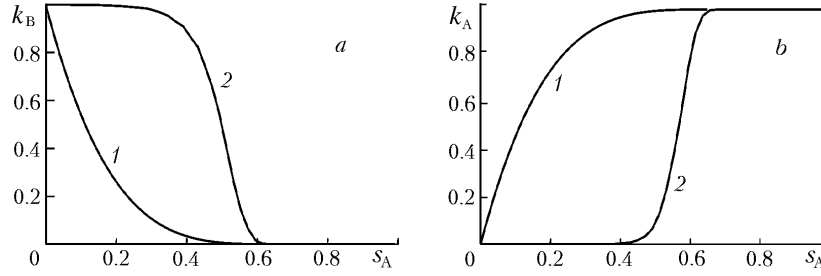
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Fig. 4. Hysteresis of the relative phase permeability of the nonwetting (a) and wetting (b) phases as a function of the wetting phase saturation s_A for the two-dimensional parameter $\zeta(\zeta_1, \zeta_2)$: 1) impregnation; 2) drainage.

$$\upsilon\,(\zeta_1, \zeta_2) = \phi \Xi^{-1} \left(\frac{\zeta_1 \zeta_2}{l^2}\right)^{3/2}, \quad \Xi = \frac{9}{16}\,\Gamma^2\left(\frac{3}{4}\right),$$

$$\sigma\,(\zeta_1, \zeta_2) = S\Theta^{-1}\left(\frac{\zeta_1 \zeta_2}{l^2}\right)\left(1 + G\left(\frac{\zeta_1 \zeta_2}{l^2}\right)^Z \exp\left(-\alpha l^{-2}\left(\zeta_1^2 + \zeta_2^2\right)\right)\right),$$

$$\Theta = \frac{\pi}{4} + G\,(1 + \alpha)^{-(3+Z)}\,\Gamma^2\left(\frac{3+Z}{2}\right),$$

(25)

where $\phi = 0.18$; $S = 46{,}476$ m^{-1} is the specific area; $l = 1.5$ μm; $\alpha = 3.3$; $G = 10^3$; $Z = 12$. The coefficients Ξ and Θ were given proceeding from the normalization conditions (2), (3). The surface tension coefficients are the same as in Example 1.

The results of the calculation of the capillary pressure and phase permeabilities are presented in Figs. 3 and 4. The kernel $K(\zeta, \zeta')$ defining the connection of pores in the space was given in a form analogous to (24):

$$K\,(\zeta_1, \zeta_2) = k_0 \Pi^{-1}\left(\frac{\zeta_1 \zeta_2}{l^2}\right), \quad \Pi = \frac{\pi}{4},$$

(26)

where $k_0 = 1$ μm^2.

The results of the modeling demonstrate a good agreement with the experimental curves of the capillary hysteresis described in [7, 10]. The form of the hysteresis is also adequate to the experiment. Note that comparing the experimental curves of the capillary hysteresis and the hysteresis of phase permeabilities to the calculation curves we can estimate the parameters characterizing the pore ensemble.

Conclusions. The basic equations of the pore ensemble model for the two-phase flow in porous media have been formulated. A simple model of the capillary hysteresis and the hysteresis of relative phase permeabilities connected with the nonmonotonicity of the function of the area-to-volume ratio of the pore has been proposed. The results of numerical calculations of the capillary pressure hysteresis demonstrate a good agreement with the experimental data described in the literature. The kind of relative phase permeabilities strongly depends on the kind of connection core K.

NOTATION

D, kernel describing the connection of pores in an elementary macrovolume; f, free energy of the mixture per unit volume; G, auxiliary coefficient; g, pore distribution function, μm^{-1}; H, total free-energy of the two-phase mixture; H_v, volume free energy; H_s, surface free energy; H^a, three-dimensional free energy flow; I_A^a, I_B^a, macroscopic flows of

the phases; i_A^a, i_B^a, three-dimensional flows of the phases along the a coordinate; j_A, j_B, inflows of phases A, B to the pore ζ from the other pores in the same elementary macrovolume; k_A, k_B, phase permeabilities; k_0, absolute permeability, μm^2; K, kernel describing the connection of pores in the space, μm^2; l, length parameter, μm; p_A, p_B, hydrostatic pressure in the phases, Pa; p_{cap}, capillary pressure, Pa; r_A, r_B, macroscopic mass densities of the phases, kg/m^3; S, specific surface, m^{-1}; s_A, s_B, macroscopic saturations of the phases A, B; t, time, sec; x, x^a, spatial Cartesian coordinates, m; α, auxiliary coefficient; Γ, gamma function; γ_A, γ_B, surface tension coefficients at the contact of the rock with the phase A and the phase B, N/m; γ_{AB}, surface tension coefficient at the contact of the phases A and B, N/m; δ, Dirac delta function; Z, auxiliary coefficient; ζ, collective parameter indexing pores in an elementary macrovolume; Θ, Ξ, Π, normalization coefficients; κ_A, κ_B, chemical potentials of the phases; μ_A, μ_B, shear viscosities of the phases, mPa·sec; $d\mu(\zeta)$, probability measure characterizing the frequency of occurrence of the pore ζ in the ensemble; ρ_A, ρ_B, mass densities of the phases, kg/m^3; Σ, free energy production; σ, pore surface area normalized to the unit volume, m^{-1}; σ_*, rms deviation; υ, pore volume normalized to the unit volume; ϕ, porosity; Ω, function characterizing the contact surface between pores; ω_A, ω_B, saturations of the phases. Subscripts: cap, capillary; A, wetting phase; B, nonwetting phase; Λ = A, B; v, volume; s, surface.

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